Hamiltonian :→ A function that is used to describe a dynamic system such as the motion of a particle in term of components of momentum and co-ordinates of space and time and that is equal to the total energy of the system when time is not explicitly part of the function.

i.e $\boxed{H = T + V}$

Hamilton's Equations :→

we have to know first Ligendre Tramformation.

if $f = f(x, y)$

if we remove x from $f(x,y)$ then according to Ligendre tranformation.

$$F_1 = f(x, y) - x \frac{\partial f}{\partial x}$$

if we romove y

$$F_2 = f(x, y) - y \frac{\partial f}{\partial y}$$

if we remove x, y both then

$$F_3 = -x \frac{\partial f}{\partial x} - y \frac{\partial f}{\partial y}$$

Now we know that:

$$L = L(q_i, \dot{q}_i, t) \rightarrow \text{Lagrangian}$$

and

$$H = H(q_i, p_i, t) \rightarrow \text{Hamiltonian}$$

if we remove $\dot{q}_i$ from the Lagrangian then it define H

$$H = L - \sum_j \dot{q}_j \frac{\partial L}{\partial \dot{q}_j}$$

but $p_j = \frac{\partial L}{\partial \dot{q}_j}$

$$H = L - \sum_j \dot{q}_j p_j$$

$$H(q_i, p_i, t) = L(q_i, \dot{q}_i, t) - \sum_i \dot{q}_i p_i$$

$$\boxed{H = L - \sum_i \dot{q}_i p_i}$$

$$\boxed{H = \sum_i p_i \dot{q}_i - L} \text{ right } \quad (1)$$

but it in general

$$L = L(q_i, \dot{q}_i, t)$$

$$dL = \sum_i \left(\frac{\partial L}{\partial q_i} dq_i + \frac{\partial L}{\partial \dot{q}_i} d\dot{q}_i + \frac{\partial L}{\partial t} dt \right)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$$

$$\frac{d}{dt}(p_i) = \frac{\partial L}{\partial q_i} \Rightarrow \dot{p}_i = \frac{\partial L}{\partial q_i}$$

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So $dL = \sum_i \dot{P}_i \, dq_i + \sum_i P_i \, d\dot{q}_i + \frac{\partial L}{\partial t} dt$.

$$dL = \sum_i \dot{P}_i \, dq_i + \sum_i d(P \dot{q}_i) - \sum_i \dot{q}_i \, dP_i + \frac{\partial L}{\partial t} dt.$$

$$dL - \sum_i d(P_i \dot{q}_i) = \sum_i \dot{P}_i \, dq_i - \sum_i \dot{q}_i \, dP_i + \frac{\partial L}{\partial t} dt.$$

$$\sum_i d(\dot{q}_i P_i) - dL = -\sum_i \dot{P}_i \, dq_i + \sum_i \dot{q}_i \, dP_i - \frac{\partial L}{\partial t} dt.$$

$$d\left(\sum_i \dot{q}_i P_i - L\right) = -\sum_i \dot{P}_i \, dq_i + \sum_i \dot{q}_i \, dP_i - \frac{\partial L}{\partial t} dt.$$

$$dH = -\sum_i \dot{P}_i \, dq_i + \sum_i \dot{q}_i \, dP_i - \frac{\partial L}{\partial t} dt \quad \text{--- (II)}$$

$$H = H(q_i, P_i, t)$$

$$dH = \sum_i \frac{\partial H}{\partial q_i} dq_i + \sum_i \frac{\partial H}{\partial P_i} dP_i + \frac{\partial H}{\partial t} dt \quad \text{--- (III)}$$

Compare equation (II) and (III)

$$-\dot{P}_i = \frac{\partial H}{\partial q_i}, \qquad \dot{q}_i = \frac{\partial H}{\partial P_i}, \qquad \frac{\partial H}{\partial t} = \frac{\partial L}{\partial t}$$

$$\boxed{\dot{q}_i = \frac{\partial H}{\partial P_i} \qquad \dot{P}_i = -\frac{\partial H}{\partial q_i}}, \quad \boxed{\frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}}$$

These are the Hamilton's equations.

$$\dot{p}_\theta = -mgl\sin\theta.$$

$$\cancel{m}l^{\cancel{2}}\ddot{\theta} = -\cancel{m}g\cancel{l}\sin\theta.$$

$$\ddot{\theta} = -\frac{g}{l}\sin\theta.$$

$$\boxed{\ddot{\theta} + \frac{g}{l}\sin\theta = 0}$$

Hamilton's Principle :→ Hamilton's principle describe the motion of the mechanical system for which all the forces (except the force of constraints) are drivable from a generalized scalar potential, which may be function of co-ordinates velocity and time, Such system are known as monogenic system. This principle is also known as Integral principle.

Hamilton's principle

$$J = \int_{x_1}^{x_2} f(y, y', x)\,dx$$

for monogenic system Hamilton's principle is define as

$$I = \int_{t_1}^{t_2} L(q, \dot{q}, t)\,dt.$$